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Investigation of heat transfer between metal and a water-cooled belt using a least square method

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Abstract—A least square method is used to determine the heat flux from a solidifying metal (steel) into a water-cooled steel belt. For this purpose, four temperatures measured in the metal are compared to those calculated using a mathematical model to solve the equation of heat conduction. The boundary conditions used in this model are given by the uppermost measured temperature and by an analytical expression for the heat-transfer coefficient from the bottom side of the metal to the cooling water. This expression involves three parameters which are determined in such a way that the square of the difference between the measured and calculated temperatures is reduced to a minimum. Minimization is achieved by using a steepest descent algorithm. A sketch of the set-up used for the experiments and two examples of temperature measurements and corresponding evaluated heat fluxes are given.

INTRODUCTION

A KNOWLEDGE of the heat flux from the solidifying metal into the mould is of interest in all kinds of casting processes. It is important for both the layout of the casting machines and for the optimization of product quality and therefore has been investigated by several authors [1–5].

The heat flux at the metal–mould interface is usually determined by measuring temperatures within the mould and/or in the solidifying metal. These data are then used to solve the so-called inverse heat conduction problem. Under conditions close to the steady state this is often easily achieved by taking two temperatures measured in the mould and the thermal conductivity of the mould material. For transient heat-flux conditions the most popular method is to solve the equation of heat conduction and to fit the calculated temperatures to those measured by trial and error variation of the heat-flux density. An alternative method is to replace the unknown boundary condition by a condition derived from the measured temperatures when the heat conduction equation is solved using a method of finite differences [6]. By using such an approach in combination with experimental data, achieved with the set-up described below, a strong oscillation of the calculated heat flux was found. A thorough examination of such effects occurring during solution of inverse heat conduction problems can be found in the work of Beck [5]. Such problems can be overcome by using least square methods [2, 3, 5, 7]. Beck [5] used an expression for the heat-flux density constant or linear in consecutive time intervals. The heat-flux densities are determined one by one for each time interval by an iterative correction procedure which is derived from the condition that an appropriate sum of square differences between measured and calculated temperatures becomes minimal. Beck's method was also used by Ho and Pehlke [2, 3]. An alternative least square technique only applicable to linear problems to determine the whole heat-flux profile at one time, which has the form of a polynomial, is described by Frank [7]. A survey of experimental and theoretical work on metal–mould heat transfer and also on methods to solve the inverse heat conduction problem is given by Ho and Pehlke [3].

With the least square method used in this paper the whole heat-flux profile, the form of which is adapted to the specific problem, is determined at one time. Minimization of the square sum is achieved by a steepest descent algorithm [8].

The method is applied to experimental data achieved with a set-up simulating solidification in belt casting processes. Such processes are for instance of great interest in the field of near net shape casting of steel [9, 10]. Heat-transfer conditions for such a type of caster were investigated by Sugitani *et al.* [11] by measuring the temperatures in the solidifying steel. Two constant heat-transfer coefficients in two successive time intervals were determined by trial and error to give the best agreement between the measured and calculated temperatures. The experimental technique described in the present paper is similar to that used by Sugitani *et al.*

ANALYSIS OF THE EXPERIMENTAL DATA

The following algorithm was used to determine the heat-flux density from a solidifying metal layer (steel) into the water-cooled belt based on temperatures measured in the metal layer.

The temperature profile in both the solid and liquid

NOMENCLATURE

- x spatial coordinate
- $x_1, \ldots, x_4$ position of the four thermocouples
- m number of temperature measurements used for the least square method
- n number of measuring positions
- t time
- T temperature
- T_c casting temperature
- T_l liquidus temperature
- T_w cooling water temperature
- T_{mc} temperature measured at the thermocouple position
- h specific enthalpy (as a function of temperature)
- h_a specific enthalpy at the previous time level in the numerical solution procedure
- a, b, c parameters in the formula for α
- $a_{opt}, b_{opt}, c_{opt}$ parameters yielding optimal fit between measured and calculated temperatures
- y_1, y_2, y_3 correction factors for a, b and c
- i integer index
- $y_{i,new}$ improved values of y_i
- f sum of square of differences between measured temperatures and those calculated using the parameters a, b and c
- F $F(y_1, y_2, y_3) = f(a \cdot y_1, b \cdot y_2, c \cdot y_3)$
- ∇F approximate gradient of F
- $(\nabla F)_i$ ith component of ∇F
- Δy increment used in the calculation of ∇F
- s step width in the steepest descent algorithm
- $\tilde{F}$ $\tilde{F}(s) = F((y_1, y_1, y_2) - s \cdot \nabla F)$
- $[1-d, 1+d] \times [1-d, 1+d] \times [1-d, 1+d]$ domain in which the minimum of F is searched, $\{(y_1, y_2, y_3)/1-d \leqslant y_i \leqslant 1+d, i = 1, 2, 3\}$
- $A_1, \ldots, A_4$, $B_1, \ldots, B_3$ coefficients in the finite difference equations.

Greek symbols

- α heat-transfer coefficient
- λ thermal conductivity (as a function of temperature)
- ρ density (as a function of temperature).

parts of the metal layer is given by the equation for heat conduction

$$\rho \frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) \tag{1}$$

if a unidirectional heat flux is assumed.

The origin of the coordinate system ($x = 0$) lies at the interface between the belt and the metal. The direction of the x-coordinate is perpendicular to the belt surface.

Let $x_1, \ldots, x_n$ be the coordinates (distances from the belt) of the thermocouples. Equation (1) is then solved in the interval $[0, x_n]$ using the following initial and boundary conditions:

$$T(x, t = 0) = T_c \tag{2}$$

$$T(x = x_n, t) = T_{mc}(x_n, t) \tag{3}$$

$$\lambda \frac{\partial T}{\partial x}(x = 0, t) = \alpha(t, a, b, c)(T(x = 0, t) - T_w). \tag{4}$$

For the heat-transfer coefficient α an expression of the form

$$\alpha(t, a, b, c) = at^b + c \tag{5}$$

is used.

If for a pure metal $T_{mc}(x_n, t)$ is equal to T_l, and both $T(x = 0, t)$ and the material properties are constant, an analytical solution (Neumann's solution) to equation (1) can be found. The heat-transfer coefficient calculated from this analytical solution for a constant surrounding temperature is in the form of equation (5) with $b = -0.5$ and $c = 0$. Parameter a is a function of material data and $T(x = 0, t)$.

Equations (1)–(5) are solved numerically using finite differences and a fully implicit enthalpy method [12]. The specific algorithm used was found to meet the requirements of accuracy and efficiency in the computing time and is outlined in the Appendix. To improve the accuracy for the first period of time the grid is significantly refined at the lower boundary ($x = 0$). For each set of parameters a, b, c, a temperature profile is obtained which is compared to the measured data calculating the square of the differences

$$f(a, b, c) = \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} 0.5[(T(x_j, t_{i+1}) - T_{mc}(x_j, t_{i+1}))^2 + (T(x_j, t_i) - (T_{mc}(x_j, t_i))^2] (t_{i+1} - t_i) \geqslant 0. \tag{6}$$

As the temperatures at the highest measuring position ($x = x_n$) are taken as the boundary condition (equation (3)) these data are not applied for the calculation of f.

To avoid complications arising due to the parameters having different orders of magnitude, their optimum values are determined using an iterative steepest descent algorithm which minimizes the function

$$F(y_1, y_2, y_3) = f(a \cdot y_1, b \cdot y_2, c \cdot y_3). \tag{7}$$

For a, b and c, values are chosen which are expected to be close to the optimum ones (e.g. $a = 1000\ \mathrm{W\ m^{-2}\ K^{-1}\ s^{-b}}$, $c = 500\ \mathrm{W\ m^{-2}\ K^{-1}}$ and $b = -0.2$ for the considered problem).

As an analytical expression for F cannot be given, an approximate gradient is calculated starting at $y_1 = y_2 = y_3 = 1$

$$\nabla F = \left(\ldots, \frac{F(\ldots, y_i + \Delta y, \ldots) - F(\ldots, y_i - \Delta y, \ldots)}{2\Delta y}, \ldots\right).$$

ith component (8)

Δy is an increment, e.g. $\Delta y = 2d/100$ with d defining the interval in which the minimum of F is expected (e.g. $d = 0.5$)

$$y_{1,2,3} \in [1-d, 1+d].$$

By looking for this minimum in the direction of steepest descent and by assuming that F is also zero there (which would mean perfect agreement between measurements and calculations), the following equation must be solved:

$$\tilde{F}(s) = F((y_1, y_2, y_3) - s\nabla F) = 0. \quad (9)$$

A first-order Taylor expansion of $\tilde{F}$ at $s = 0$ yields

$$s \approx \frac{F(y_1, y_2, y_3)}{(\nabla F)^2}. \quad (10)$$

The new point is then calculated according to

$$y_{i,\mathrm{new}} = \begin{cases} 1+d & \text{if} \quad y_i - s\cdot(\nabla F)_i \geqslant 1+d \\ 1-d & \text{if} \quad y_i - s\cdot(\nabla F)_i \leqslant 1-d \\ y_i - s\cdot(\nabla F)_i & \text{otherwise.} \end{cases} \quad (11)$$

If

$$F((y_1, y_2, y_3)_{\mathrm{new}}) < F(y_1, y_2, y_3) \quad (12)$$

the procedure continues with equation (8) by calculating a new gradient at $(y_1, y_2, y_3)_{\mathrm{new}}$, otherwise s is multiplied by 0.5 and $y_{i,\mathrm{new}}$ is calculated again using this modified value of s and equation (11).

The iteration is continued until F has become sufficiently small or until no further improvement can be achieved (for the used values of a, b, c and d). If (y_1, y_2, y_3) finally lies on the boundary of $[1-d, 1+d] \times [1-d, 1+d] \times [1-d, 1+d]$ the iteration has to be restarted with modified values for a, b, c and d.

The optimum parameters for the calculation of α according to equation (5) are finally given by

$$a_{\mathrm{opt}} = a \cdot y_1$$

$$b_{\mathrm{opt}} = b \cdot y_2$$

$$c_{\mathrm{opt}} = c \cdot y_3. \quad (13)$$

The described method was tested by calculating the temperatures at four different positions using Neumann's solution and adopting them as 'measured' data. A comparison of the heat-transfer coefficient, determined using the method described, with that achieved from the analytical solution for the temperature profile gave an almost perfect agreement. Of course, as already mentioned, in this case the analytical expression for α has exactly the same form as equation (5).

EXPERIMENTAL SET-UP

The experimental set-up is sketched in Fig. 1. The liquid steel is poured into a cylinder standing on the belt (steel, 1.5 mm thick). The belt is water-cooled from below.

The cylinder consists of an outer metal cylinder and an inner one of refractory material ('insert' in Fig. 1). In between there is an insulation layer. The inner cylinder is preheated before the experiment to prevent heat losses to the sides and, therefore, to ensure the mainly axial heat flux assumed in the mathematical model.

The temperature in the steel is measured by four thermocouples which are positioned a few millimetres away from the belt on the centreline of the cylinder. The thermocouples are inserted into protection tubes (Al_2O_3, 1 mm in diameter).

As the protection tubes sometimes break and lift up somewhat, the small ingots are cut in the longitudinal direction after completion of the experiment to determine the actual position of the thermocouples. The symbols in Fig. 2 show for two examples the temperatures measured at the four positions as a function of time. The distance between the thermocouple and the belt is also given for each curve. In the example given in Fig. 2(a) the thermocouples were practically in their original position, whereas Fig. 2(b) shows a significant shift. For the heat-flux analysis described, a shift in the thermocouple position causes no problem provided that the final position is known.

As the thermocouples must be heated up from room temperature, when they come into contact with the steel it takes about 2 s before stable data are achieved. The temperature at $t = 0$ is that measured in the melt.

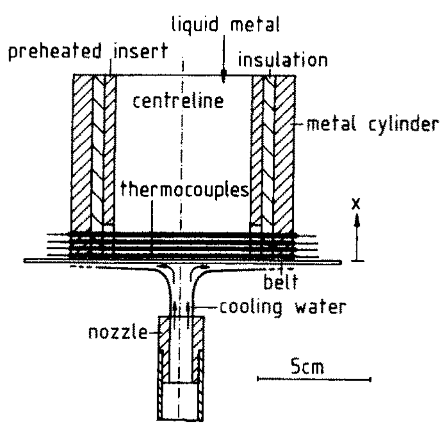

FIG. 1. Sketch of the experimental set-up.

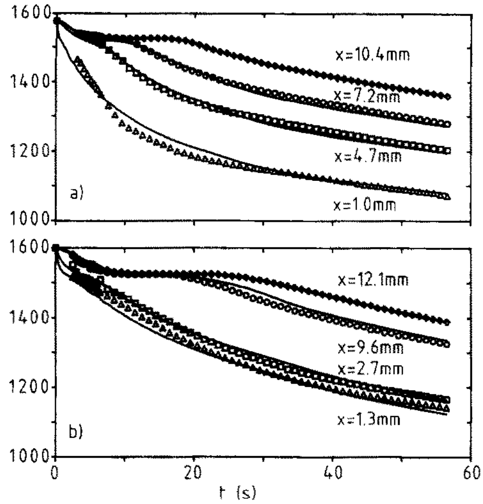

FIG. 2. Two examples for temperatures measured in the solidifying steel (symbols). The lines represent temperatures calculated with the 'optimal' parameters in the formula for the heat-transfer coefficient.

RESULTS

An analysis of the measured data, given by the symbols in Fig. 2 leads to the results shown in Fig. 3. The calculated bottom temperature ($x = 0$) of the steel layer, the heat-flux into the belt and the heat-transfer coefficient, which is calculated from the optimum parameters, are plotted as a function of time.

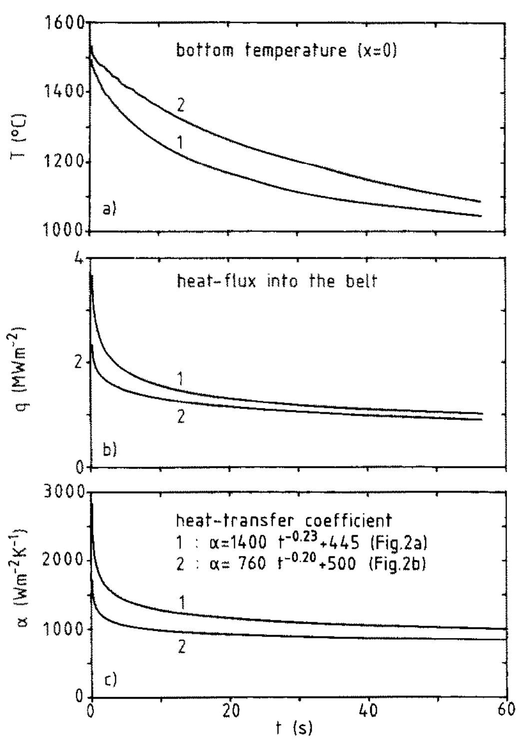

FIG. 3. Calculated bottom temperatures, heat-flux densities and heat-transfer coefficients for the two examples in Fig. 2.

The heat flux is achieved directly from the bottom temperature and the heat-transfer coefficient, which is for a cooling water temperature $T_w = 30°C$. The optimum parameters inserted into the formula for α are also given in Fig. 3(c). Both the heat-flux density into the belt and the heat-transfer coefficient decrease strongly at the first moment indicating that the developing contact resistance between the forming solid layer and the belt is of great importance. In a series of experiments in which the belt was made of stainless steel in place of the non-alloyed steel used in the presented examples, the relatively low thermal conductivity sometimes leads to a sticking of the small ingot at the belt. Under such irregular conditions the good contact results in a significantly increased heat-transfer coefficient confirming the importance of the contact resistance.

The temperatures calculated at the measuring positions in the steel layer using the optimal α-profile are presented by the lines in Fig. 2 showing good agreement.

The heat-transfer coefficients are of similar magnitude to those reported by Sugitani *et al.* [11]. They found good agreement between measurements and calculations, choosing a ≈ 3000 W m^{-2} K^{-1} for the initial 2 s and afterwards $\alpha \approx 1200$ W m^{-2} K^{-1}. The heat flux into the belt is also quite similar both in magnitude and profile to that measured for the broad sides of moulds for continuous casting of steel slabs [13] if the distance from the meniscus is converted into a corresponding time by dividing by the casting speed.

A relatively large scattering was found in the results for different experiments. This may be due to an insufficient control during the experiments of the flow field in the steel after manual pouring and also of other parameters such as reoxidation. Different types of structures for the bottom surfaces of the small ingots were found to develop and their roughness has a significant effect on heat transfer.

The computing time required to run the written program in order to achieve the presented examples is in the range of about 10 min using a modern super mini computer.

CONCLUSIONS

A technique was tested to determine heat-flux densities from temperatures measured in a solidifying metal layer. This technique is based on a least square fit between the measured and calculated data and was found to be accurate on data derived using an analytical solution and also insensitive to certain fluctuations of measured data or to a lack of data during the first seconds. No stability problems found when testing other methods or reported by other authors arose. The advantage of the technique presented over a pure trial and error approach in fitting measured and calculated data is obvious. Of course, the results obtained depend to a certain extent on the expression chosen for the heat-transfer coefficient. There are, however, no principal restrictions, such as continuity or number of parameters involved, on the form of this expression.

Application of the technique described to the solidification of steel on a water-cooled belt leads to heat-flux density profiles similar to those measured in moulds during the continuous casting of steel slabs indicating that in both processes the mechanisms controlling the heat transfer are similar.

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APPENDIX

Using finite differences and an implicit scheme, discretization of equation (1) gives a system of equations each having the form

$$A_1(i)T(i-1)+A_2(i)T(i)+A_3(i)T(i+1)+A_4(i)h(i) = h_a(i) \tag{A1}$$

where i is the current number of an internal grid point. $h_a(i)$ is the known specific enthalpy at grid point i for the previous time level.

To obtain an equation for the variable h only, equation (A1) can be rewritten to give

$$B_1(i)h(i-1)+B_2(i)h(i)+B_3(i)h(i+1) = h_a(i) \tag{A2}$$

with

$$B_1(i) = A_1(i)\frac{T(i-1)}{h(i-1)} \tag{A3a}$$

$$B_2(i) = A_2(i)\frac{T(i)}{h(i)}+A_4(i) \tag{A3b}$$

$$B_3(i) = A_3(i)\frac{T(i+1)}{h(i+1)}. \tag{A3c}$$

The system of equations (A2), which is completed by two equations for the grid points on the boundary derived from the boundary conditions, is solved by iteration in the following way. For the first iteration cycle the temperature from the previous time level is taken to calculate the material data (λ, ρ) entering $A_1, \ldots, A_4$. The ratio T/h at the grid points is also taken from the previous time level. This enables B_1, B_2 and B_3 to be calculated and the linearized system (A2) to be solved, yielding new enthalpies h at the grid points. Using the known h–T relationship new temperatures, material data, values for the T/h ratio and, therefore, new values for B_1, B_2, B_3 are achieved. This procedure is repeated until a prescribed stability for h and T is reached.

ETUDE PAR UNE METHODE DE MOINDRE CARRE DU TRANSFERT THERMIQUE ENTRE UN METAL ET L'EAU DE REFROIDISSEMENT

Résumé—Une méthode de moindre carré est utilisée pour déterminer le flux thermique partant d'un métal en solidification (acier) dans une bande refroidie par eau. Pour cela quatre températures mesurées dans le métal sont comparées à celles calculées à partir d'un modèle mathématique pour résoudre l'équation de la conduction thermique. Les conditions aux limites utilisées dans ce modèle sont données par la température mesurée la plus élevée et par une expression analytique pour le métal et l'eau de refroidissement. Cette expression contient trois paramètres qui sont déterminés de façon à réduire à l'extrême le carré de la différence entre la température mesurée et celle calculée. La minimisation est obtenue en utilisant un algorithme de plus grande pente. Un schéma du montage utilisé par les expériences est donné ainsi que deux exemples de mesure de température et d'évaluation correspondante de flux thermique.

UNTERSUCHUNG DES WÄRMEÜBERGANGS ZWISCHEN METALL UND EINEM WASSERGEKÜHLTEN BAND DURCH FEHLERQUADRATMINIMIERUNG

Zusammenfassung—Ein Minimierungsverfahren wird benutzt, um den Wärmestrom zwischen einer erstarrenden Stahlschicht und einem wassergekühlten Stahlband zu untersuchen. Hierzu werden vier im Stahl gemessene Temperaturen mit numerisch berechneten verglichen. Als Randbedingungen werden dabei die gemessene Temperatur am obersten Thermoelement sowie ein Wärmeübergangskoeffizient von der Unterseite der Stahlschicht zum Kühlwasser benutzt. In die Formel für diesen gehen drei Parameter ein, die so bestimmt werden, daß die quadratische Abweichung zwischen berechneten und gemessenen Temperaturen minimal wird. Für die Minimierung wird ein Gradientenverfahren angewandt. Der benutzte experimentelle Aufbau wird skizziert, und es werden Beispiele für durchgeführte Auswertungen angegeben.

ИССЛЕДОВАНИЕ ТЕПЛОПЕРЕНОСА МЕЖДУ МЕТАЛЛОМ И ВОДООХЛАЖДАЕМЫМ ПОЯСОМ С ИСПОЛЬЗОВАНИЕМ МЕТОДА НАИМЕНЬШИХ КВАДРАТОВ

Аннотация—Методом наименьших квадратов обрабатываются результаты по теплопереносу от затвердевающего металла (стали) к водоохлаждаемому стальному поясу. С этой целью четыре значения измеренных в металле температуры сравниваются с рассчитанными на основе математической модели из решения уравнения теплопроводности. Граничные условия, используемые в данной модели, записываются на основании максимальной измеренной температуры и аналитического выражения для коэффициента теплообмена от нижней части металла к охлаждающей воде. Полученное выражение включает три параметра, определяемые минимизацией квадрата разности между измеренной и рассчитанной температурами. Минимизация достигается при помощи алгоритма быстрейшего спуска. Приводятся схема экспериментальной установки, а также два бримера измерений температуры и соответствующих рассчитанных тепловых потоков.